Formal Languages and Automata Theory
TDDD14
Assignment 2

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1. Start by constructing a grammar for $h(L)$:

$$\begin{aligned} S &\rightarrow ASB \mid ab \\ A &\rightarrow a \\ B &\rightarrow b \mid bb \end{aligned}$$

The role of the first production, $S \rightarrow ASB$, is to produce an equal amount of the nonterminals A and B in two groups, like $A^n B^n$. The nonterminal A always results in exactly one terminal a. The nonterminal B results in one or two terminal b. Therefore, the first production generates strings x where $0 < \#a(x) \leq \#b(x) \leq 2 \cdot \#a(x)$. The remaining production is $S \rightarrow ab$, which serves to transform the last inequality into a strict one,

$$0 < \#a(x) + 1 \leq \#b(x) + 1 < 2 \cdot (\#a(x) + 1) = 2 \cdot [\#a(x) + 1] + 1,$$

or simpler,

$$0 < \#a(x) \leq \#b(x) < 2 \cdot \#a(x).$$

Next, add a new nonterminal C with productions which generate arbitrarily long strings consisting only of the terminal c, that is, c^*.

$$\begin{aligned} S &\rightarrow ASB \mid aCb \\ A &\rightarrow Ca \\ B &\rightarrow bC \mid bCbC \\ C &\rightarrow CC \mid c \mid \varepsilon \end{aligned}$$

This is the sought context-free grammar for the language L.

If the grammar was not ambiguous, there would be no case where two different leftmost derivations could generate the same string. Consider a leftmost derivation of the string $aaabbbb$.

$$S \rightarrow ASB \rightarrow aSB \rightarrow aASBB \rightarrow aaSBB \rightarrow aaabBB \qquad (1)$$

Continuing from (1), two leftmost derivations of $aaabbbb$ are possible:

$$\begin{aligned} aaabBB &\rightarrow aaabbbB \rightarrow aaabbbb \\ aaabBB &\rightarrow aaabbB \rightarrow aaabbbb \end{aligned}$$

Therefore, by contradiction, the grammar must be ambiguous.