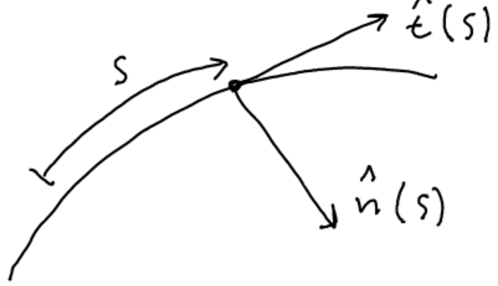
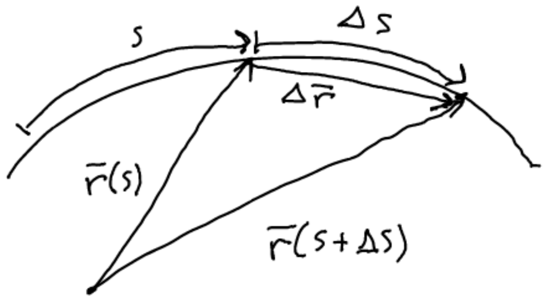
Föreläsning 1, Mekanik 1.

Def: Tangentvektor $\hat{t} = \hat{t}(s)$ och normalvektor $\hat{n} = \hat{n}(s)$ för en plan kurva:





$$\hat{t} = \frac{d\bar{r}}{ds} \qquad (1)$$

$$\hat{n} = \frac{d\hat{t}}{ds} \Big/ \left|\frac{d\hat{t}}{ds}\right| \qquad (2)$$

Def: Krökning $\varkappa = \varkappa(s)$:

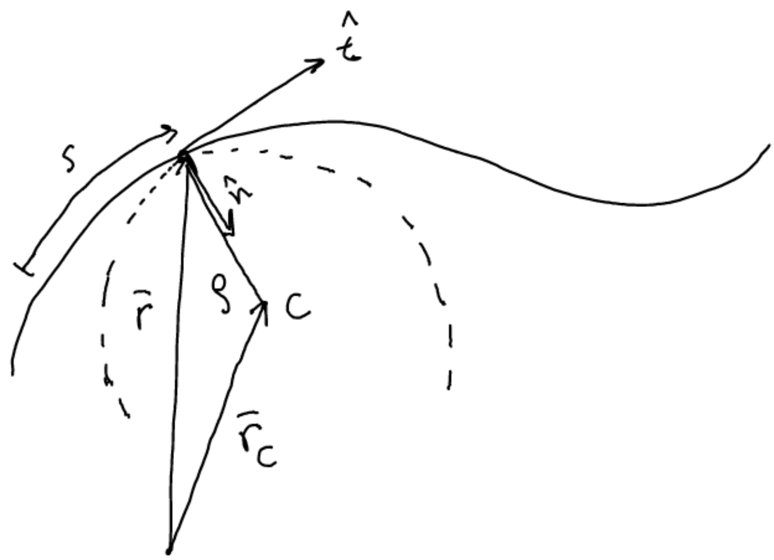
$$\varkappa = \left|\frac{d\hat{t}}{ds}\right| = \left|\frac{d^2\bar{r}}{ds^2}\right| \qquad (3)$$

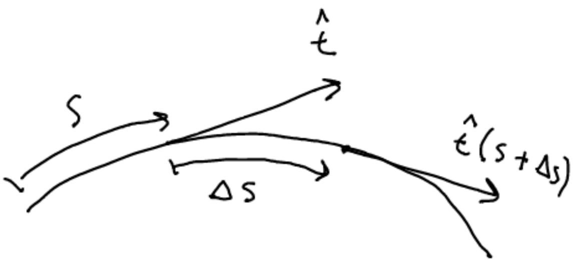
<u>Sats</u>: Krökning $\varkappa$ för godtycklig parameter u:

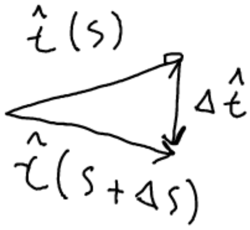
$$\varkappa = \frac{\left|\frac{d\bar{r}}{du} \times \frac{d^2\bar{r}}{du^2}\right|}{\left|\frac{d\bar{r}}{du}\right|^3}$$

Bevis: Se kompendiet.

Def: Oskulerande cirkeln:







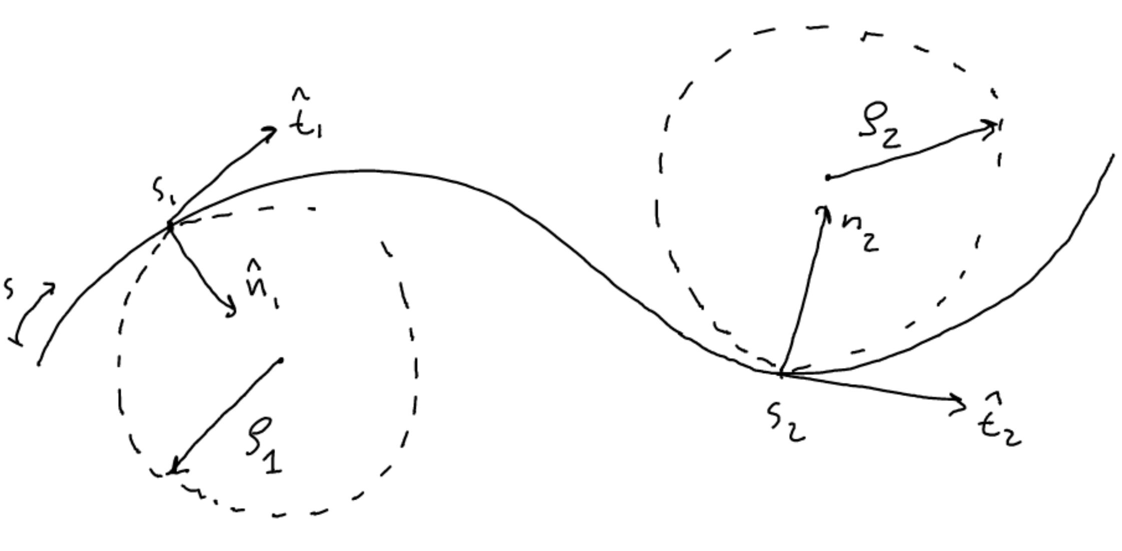
$$\hat{n}(s) = \frac{\frac{d\hat{t}(s)}{ds}}{\left|\frac{d\hat{t}(s)}{d\hat{t}(s)}\right|}$$

Radie (krökningsradie) ρ:

$$\rho = \frac{1}{\varkappa} \qquad (4)$$

Centrum (krökningscentrum) C

$$r_c = \bar{r} + \rho\hat{n} \qquad (5)$$

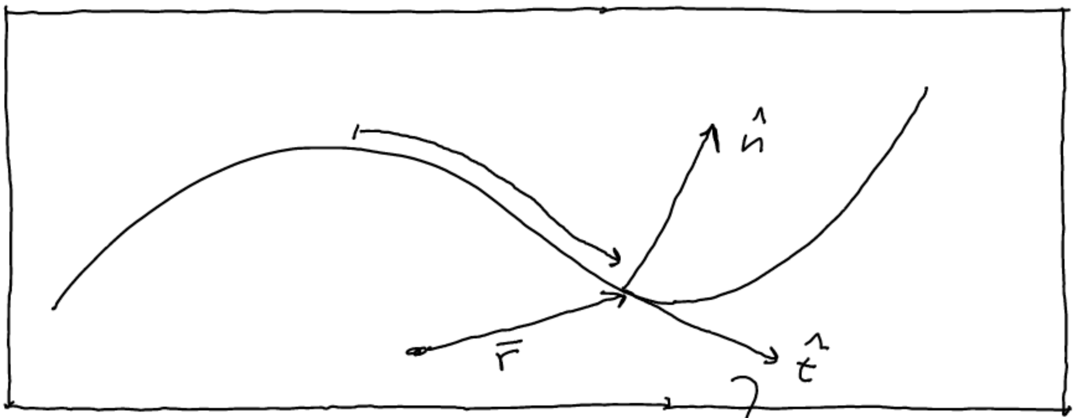


Sats!

Den oskulerande cirkeln är den cirkel som bäst approximerar kurvan i en given punkt.

Bevis: Se kompendiet.

Hastighet och acceleration i naturliga basen (tn)



$\bar{r} = \bar{r}(s(t))$.

inertialram, Dvs fix referensram.

Hastighet:

$$\bar{v} = \underbrace{\frac{d\bar{r}}{dt}}_{\dot{\bar{r}}} = \underbrace{\frac{d\bar{r}}{ds}}_{=\hat{t} \text{ enl } (1)} \cdot \underbrace{\frac{ds}{dt}}_{\dot{s}} \Rightarrow \quad \bar{v} = \underbrace{\dot{s}}_{v}\,\hat{t},$$

Fart:

$$|\bar{v}| = |\dot{s}| = |v|.$$

Acceleration:

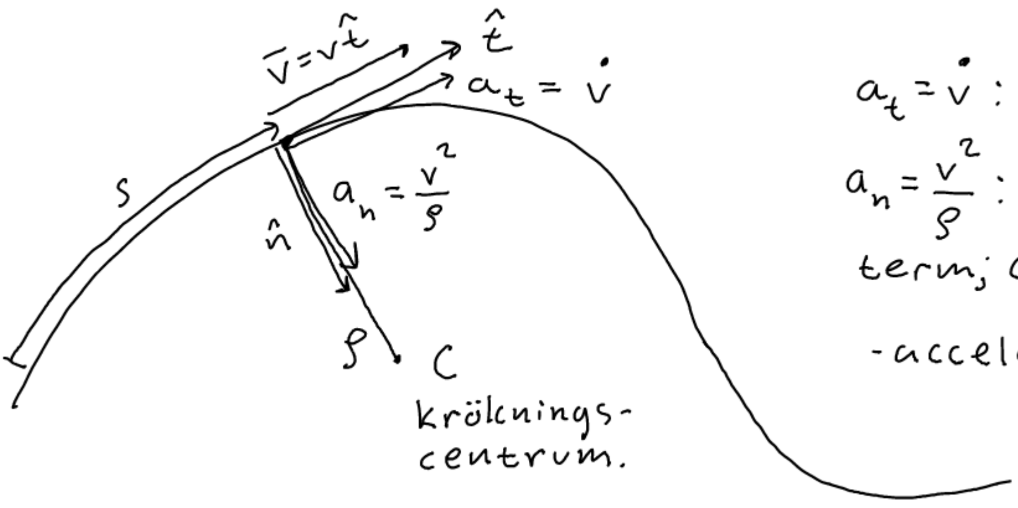
$$\bar{a} = \dot{\bar{v}} = \dot{v}\hat{t} + v\dot{\hat{t}} = \dot{v}\hat{t} + v\underbrace{\frac{d\hat{t}}{ds}}_{\hat{n}\varkappa} \cdot \underbrace{\frac{ds}{dt}}_{\dot{s}=v} = \dot{v}\hat{t} + \varkappa\hat{n}\cdot v^2 = \left[\varkappa = \frac{1}{\rho}\right] =$$

(2)

$$= \dot{v}\hat{t} + \frac{v^2}{\rho}\hat{n}.$$

$$\boxed{\bar{a} = \dot{v}\hat{t} + \frac{v^2}{\rho}\hat{n}}$$

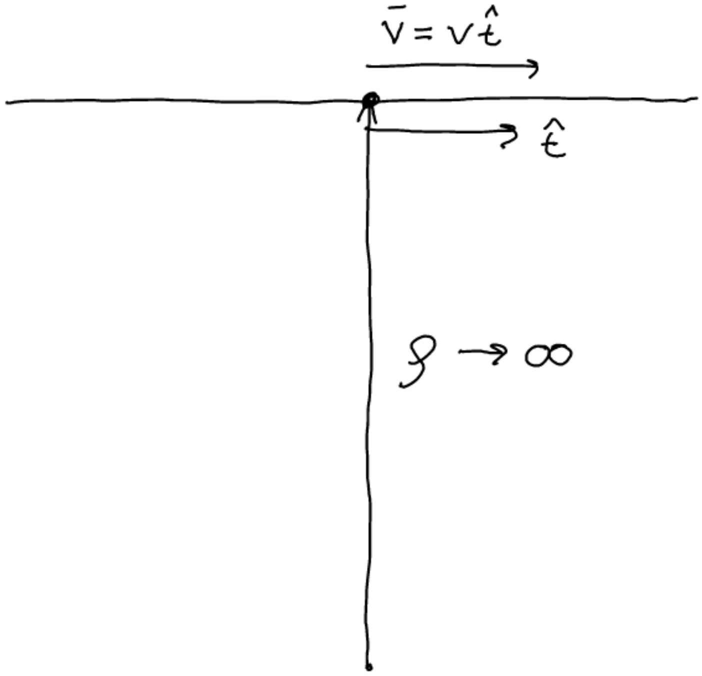
ρ krökningsradien.



$a_t = \dot{v}$: fartändringsterm.

$a_n = \frac{v^2}{\rho}$: riktningsändringsterm; centripetal-acceleration.

<u>Ex</u>: Rätlinjig rörelse.

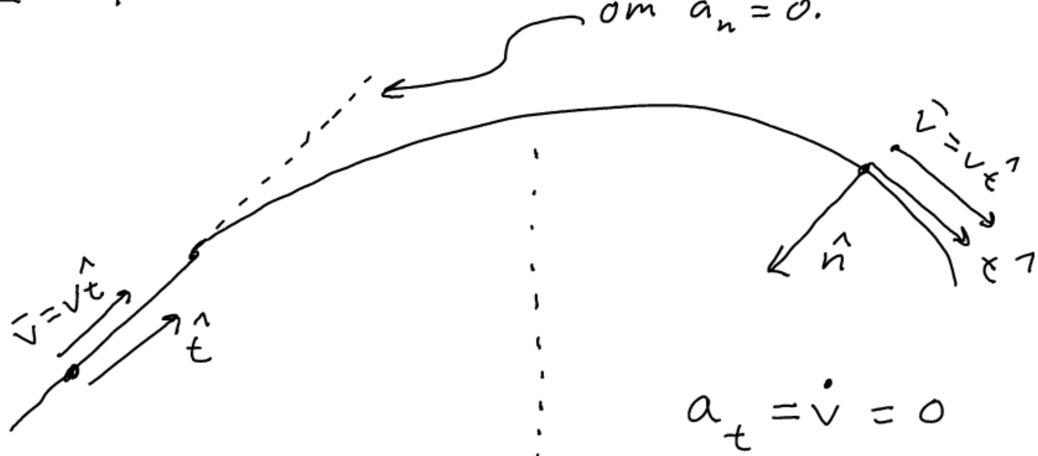


$\rho \to \infty \Rightarrow a_n \to 0$

$$\therefore \quad \vec{a} = \underbrace{\dot{v}}_{a = a_t}\hat{t}$$

∎

Ex: $|\bar{v}|$ konstant.



$a_t = \dot{v} = 0$

$a_n = 0$ ty $\rho \to \infty$

$\therefore \; \bar{a} = 0$

$a_t = \dot{v} = 0$

$a_n = \dfrac{v^2}{\rho}$

$a \neq 0$.

OBS: Krökt bana $\Rightarrow$ $\bar{a} \neq \bar{0}$!

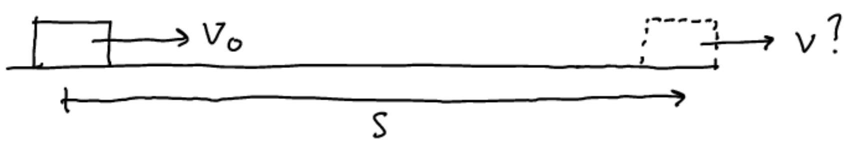
■

$$\left.\begin{aligned} v &= \frac{ds}{dt} \\ a_t &= \dot{v} = \frac{dv}{dt} \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} dt &= \frac{ds}{v} \\ dt &= \frac{dv}{a_t} \end{aligned}\right. \Rightarrow$$

Varginons Sats:

$$\boxed{a_t\, ds = v\, dv}$$

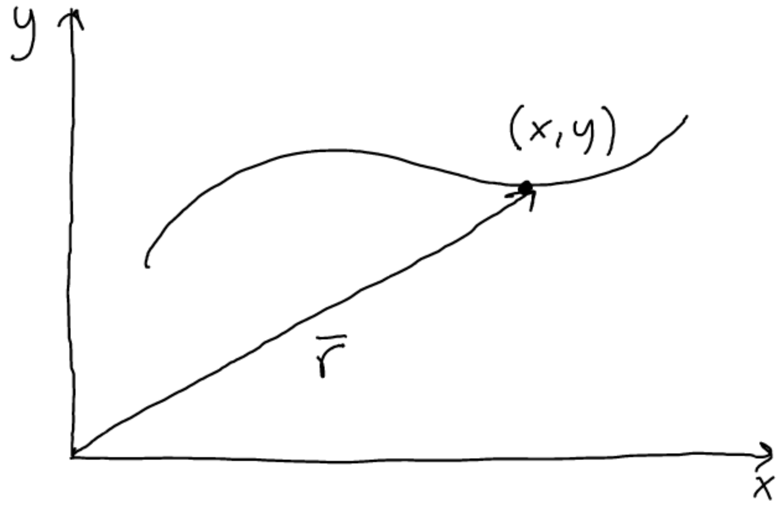
Ex



$a = a_t$ konstant. $\Rightarrow$

$$\int_0^s a\,ds = \int_{v_0}^{v} v\,dv \iff as = \frac{1}{2}\left(v^2 - v_0\right) \Rightarrow v = \sqrt{v_0^2 + 2as} \qquad \blacksquare$$

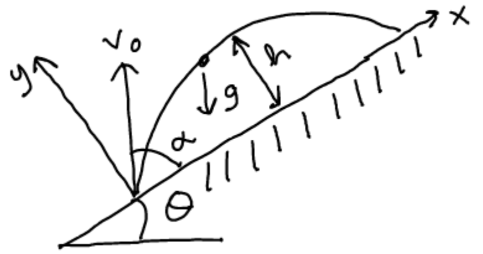
Kartesiska koordinater.



$$\bar{r} = x\hat{x} + y\hat{y}$$
$$\bar{v} = \dot{\bar{r}} = \dot{x}\hat{x} + \dot{y}\hat{y}$$
$$\bar{a} = \dot{\bar{v}} = \ddot{x}\hat{x} + \ddot{y}\hat{y}$$

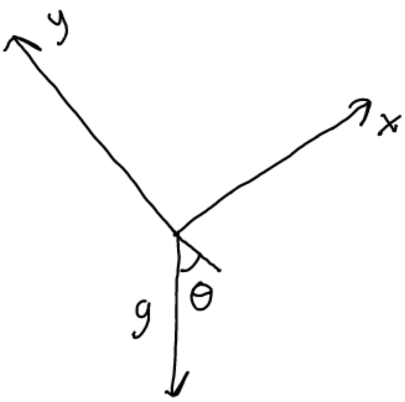
$$\dot{x} = \frac{dx}{dt}, \quad \ddot{x} = \frac{d\dot{x}}{dt} \Rightarrow \boxed{\begin{aligned} \ddot{x}\,dx &= \dot{x}\,d\dot{x} \\ \ddot{y}\,dy &= \dot{y}\,d\dot{y} \end{aligned}} \qquad (6)$$

Ex:



Givet: $\bar{a} = g, \downarrow$

Sökt: h

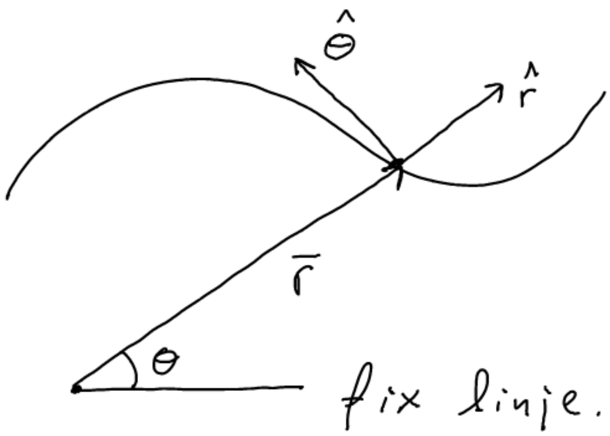


$h = y_{max}$

$$(6) \Rightarrow \int_0^h \ddot{y}\,dy = \int_{v_0 \sin\alpha}^{0} \dot{y}\,d\dot{y}$$

$$\Rightarrow -gh\cos\theta = \left[\frac{1}{2}\dot{y}^2\right]_{v_0\sin\alpha}^{0} \Rightarrow h = \frac{v_0^2 \sin^2\alpha}{2g\cos\theta} \qquad \blacksquare$$

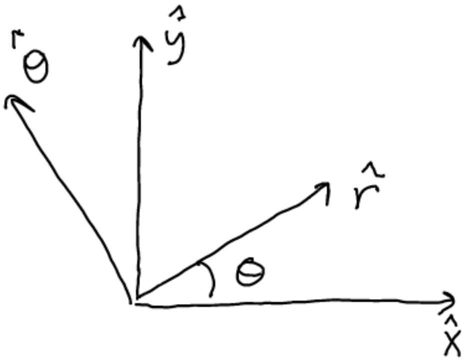
Polära koordinater



$\bar r = r\hat r$

$\bar v = \dot r\hat r + r\dot{\hat r}$

$\dot{\hat r}$?



$\hat r = \cos\theta\,\hat x + \sin\theta\,\hat y$

$\hat\theta = -\sin\theta\,\hat x + \cos\theta\,\hat y$

$\therefore\quad \dot{\hat r} = -\sin\theta\,\dot\theta\,\hat x + \cos\theta\,\dot\theta\,\hat y = -\dot\theta\hat r$

$$\boxed{\bar v = \dot r\hat r + r\dot\theta\hat\theta}$$

($\dot\theta$ i rad/s)

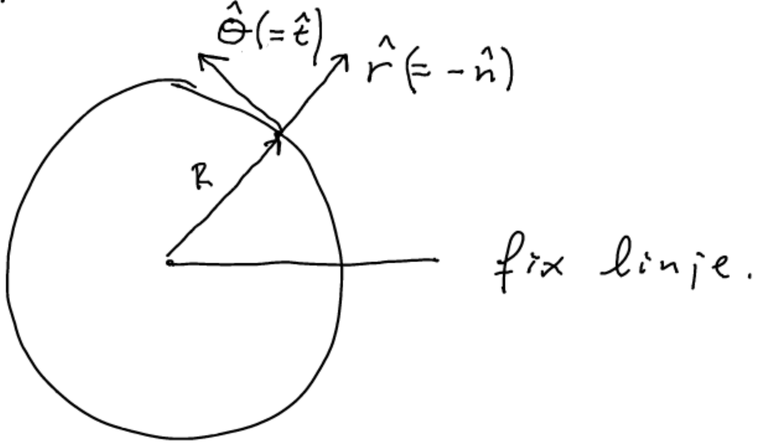
$$\bar a = \dot{\bar v} = \ddot r\hat r + \dot r\dot{\hat r} + \dot r\dot\theta\hat\theta + r\ddot\theta\hat\theta + r\dot\theta\dot{\hat\theta} = \ddot r\hat r + 2\dot r\dot\theta\hat\theta + r\ddot\theta\hat\theta - r\dot\theta^2\hat r$$

$$\boxed{\bar a = \left(\ddot r - r\dot\theta^2\right)\hat r + \left(r\ddot\theta + 2\dot r\dot\theta\right)\hat\theta}$$

$(6) \Rightarrow \quad \ddot r\,dr = \dot r\,d\dot r$

$\ddot\theta\,d\theta = \dot\theta\,d\dot\theta$

Exempel: Cirkelrörelse

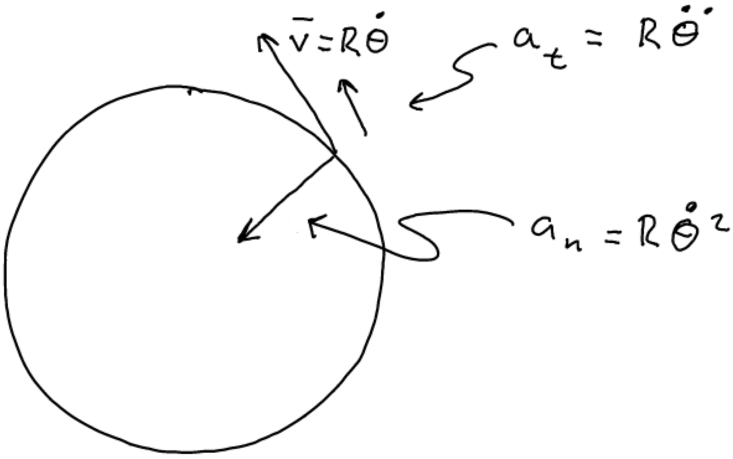


$r = R$ konstant

$\bar{v} = R\dot{\theta}\hat{\theta}$

$\bar{a} = (-R\dot{\theta}^2)\hat{r} + (R\ddot{\theta})\hat{\theta}$

<u>Alt</u>: Naturliga basen.



$\bar{v} = R\dot{\theta}\hat{t}$

$\bar{a} = R\ddot{\theta}\hat{t} + R\dot{\theta}^2\hat{n}$