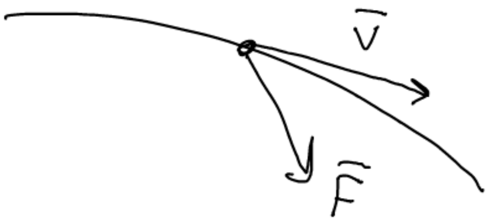
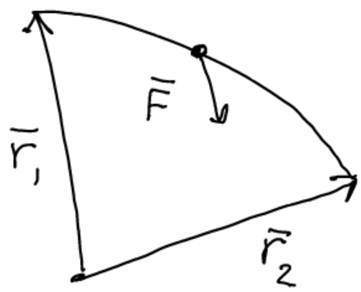
Föreläsning 5, mekanik del 1.

effekt $\underline{P}$ av kraft $\bar{F}$



$$\boxed{P = \bar{F} \cdot \bar{v}}$$

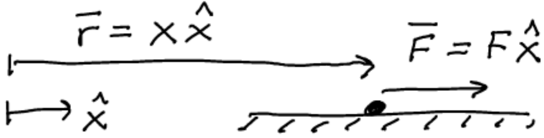
Arbete U uträttat av F under $t_1 \leq t \leq t_2$.



$$U = \int_{t_1}^{t_2} \underline{P}\, dt = \int \bar{F} \cdot d\bar{v}\, dt = \int \bar{F} \cdot \frac{d\bar{r}}{dt}\, dt = \int \bar{F} \cdot d\bar{r}.$$

$$U = \int_{\bar{r}_1}^{\bar{r}_2} \bar{F} \cdot dr.$$

Ex:

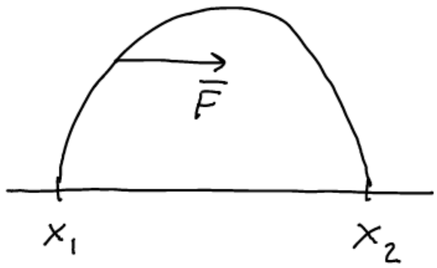


, $\bar{F}$ konst.

$$U = \int F \cdot d\bar{r} = \int F\hat{x} \cdot dx\hat{x} =$$

$$= F\int_{x_1}^{x_2} dx = F(x_2 - x_1)$$

"Kraft * vägen i kraftens riktning"



Arbete-energilagen

$$\bar{F} = m\bar{a} \Rightarrow \underbrace{\bar{F} \cdot \bar{v}}_{P_{tot}} = \underbrace{m\bar{a} \cdot \bar{v}}_{\frac{d}{dt}\left(\underbrace{m\frac{v \cdot v}{2}}_{T}\right)}$$

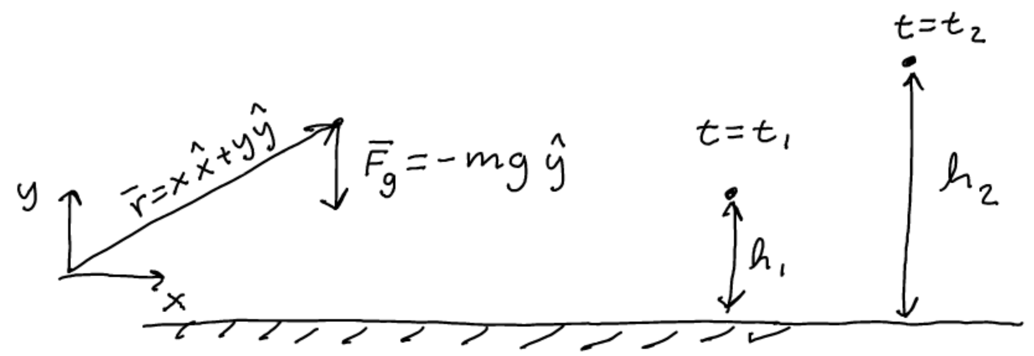
$$\therefore \quad P_{tot} = \dot{T} \quad (1)$$

$$\boxed{T = \frac{mv^2}{2}}$$ Rörelseenergi

$$(1) \Rightarrow \underbrace{\int_{t_1}^{t_2} \underline{P}_{tot}\, dt}_{U_{tot}} = \underbrace{\int_{t_1}^{t_2} \frac{dT}{dt}\, dt}_{\underbrace{T(t=t_2)}_{T_2} - \underbrace{T(t=t_1)}_{T_1}}$$

$$\therefore \quad U_{tot} = \Delta T \quad (2) \,, \quad \Delta T = T_2 - T_1$$

Arbetet av tyngdkraft.



$$U_g = \int \bar{F}_g \cdot d\bar{r} = \int (-mg\,\hat{y}) \cdot (dx\,\hat{x} + dy\,\hat{y})$$

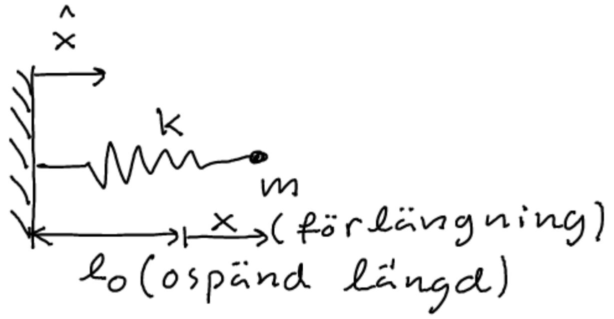
$$= \int_{h_1}^{h_2} -mg\, dy = -mg(h_2 - h_1)$$

Inför lägesenergin V_g

$$\boxed{V_g = mgh}$$

$$\therefore \quad U = -\underbrace{(V_{g_2} - V_{g_1})}_{\Delta V_g} \qquad (3)$$

Arbetet av fjäderkraft



$$\bar{r} = (l_0 + x)\hat{x}$$

Frilägg

$$\bar{F}_{fj} = -kx\hat{x}$$

$$U_{fj} = \int \bar{F}_{fj} \bullet d\bar{r} = \int_{x_1}^{x_2} (-kx\hat{x}) \bullet dx\,\hat{x} = \frac{k}{2}(x_2^2 - x_1^2)$$

Inför fjäderenergin

$$\boxed{V_e = \frac{k}{2}x^2}$$

$$\therefore \; U_{fj} = -\underbrace{(V_{e_2} - V_{e_1})}_{\Delta V_e} \qquad (4)$$

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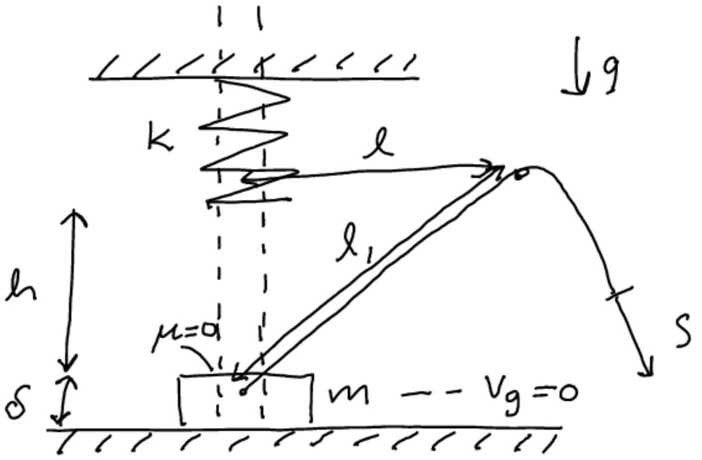
(3) & (4) i (2) $\Rightarrow$

$$(-V_g) + (-V_e) + U = \Delta T \Leftrightarrow$$

$$\boxed{U = \Delta T + \Delta V_g + \Delta V_e} \quad (5)$$, Arbete-energilagen.

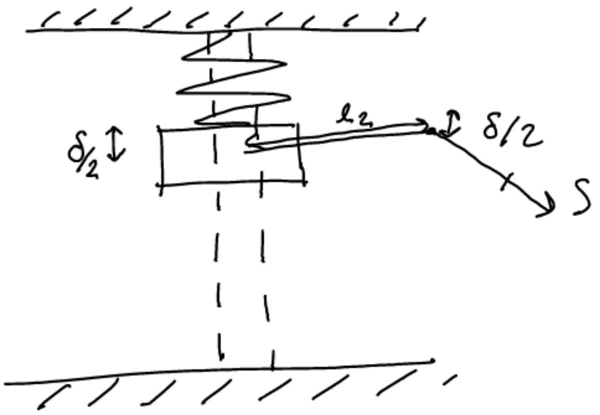
U: Arbetet av alla krafter utom tyngd och fjäderkrafter.

53)



Sökt: k så max ihoptryckning $= \delta/4$

Slutläge:



$$U = \Delta T + \Delta V_g + \Delta V_e \quad (1)$$

Läge 1 (startläge):

$T_1 = 0$

$V_{g_1} = 0$

$V_{e_1} = 0$

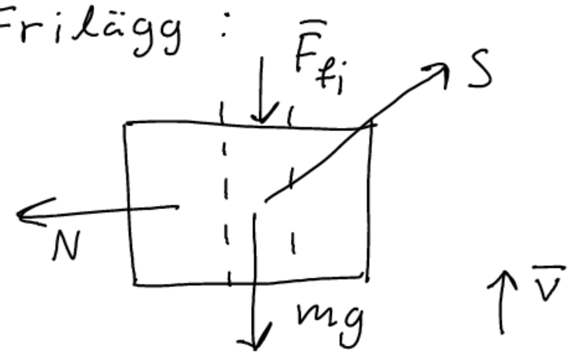
Läge 2 (slutläge)

$T_2 = 0$ (hastigheten $=0$ vid maximal hoptryckning)

$$V_{g_2} = mg\left(h + \frac{\delta}{4}\right)$$

$$V_{e_2} = \frac{k}{2}\left(\frac{-\delta}{4}\right)^2 = \frac{k\delta^2}{32}$$

Frilägg:



$$\mathcal{U} = \mathcal{U}_N + \mathcal{U}_S$$

$$N \cdot \bar{v} = 0 \Rightarrow \mathcal{U}_N = 0.$$

$\mathcal{U} = \mathcal{U}_S = S \cdot$ "utdragen snörlängd" $= S(l_1 - l_2)$

$$= S\left(\sqrt{l^2 + (h + \delta/2)^2} - \sqrt{l^2 + \left(\frac{\delta}{4}\right)^2}\right)$$

Ins i (1) $\Rightarrow$

$$S(l_1 - l_2) = mg\left(h + \frac{\delta}{4}\right) + \frac{k\delta}{32}$$

$$\Leftrightarrow k = \frac{32}{\delta^2}\left[S(l_1 - l_2) - mg\left(h + \frac{\delta}{4}\right)\right]$$

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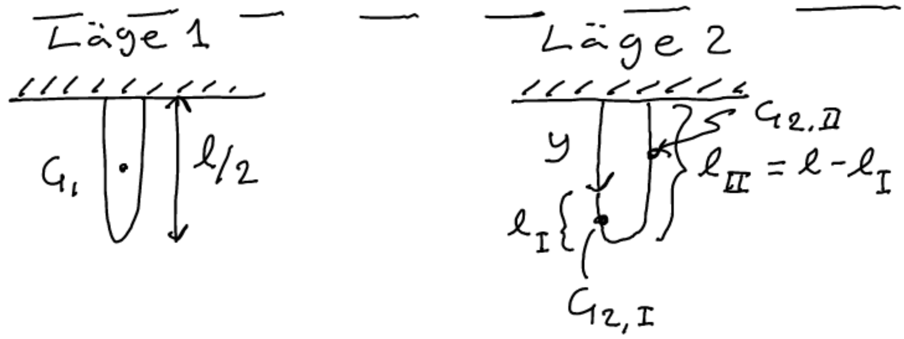
55)



Givet:
ρ (kg/m)

sökt
$v = \dot{y}$

$$\underbrace{\mathcal{U}}_{=0} = \Delta T + \Delta V_g + \underbrace{\Delta V_e}_{=0} \qquad (1)$$

Läge 1

$T_1 = 0$

$V_{g_1} = -\underset{\rho l}{\underbrace{m}} g \frac{l}{4}$

Läge 2

$T_2 = \frac{m_I v^2}{2} = \frac{l_1 \rho v^2}{2}$

$V_g = -\rho g l_I g \left(y + \frac{l_I}{2}\right) - \rho l_2 g \frac{l_{II}}{2}$

l_I, l_{II} ?

fig $\Rightarrow$ $2(y + l_I) = l + y = l_I = \frac{l}{2} - \frac{y}{2}$

$l_{II} = l - l_I = \frac{l}{2} + \frac{y}{2}$

$\therefore \; V_{g_2} = \dots = \frac{-\rho g}{2}\left[\frac{-1}{2} y^2 + ly + \frac{l^2}{2}\right]$

Ins i (1) $\Rightarrow$

$0 = \frac{\rho}{4}(l - y) v^2 - \frac{\rho g y}{4}(-y + 2l)$

$\Rightarrow \; v = \sqrt{\frac{gy(2l - y)}{l - y}}$