Tentamen i Vågfysik
NFYB01, TFYA10, TFYA59
2012-05-31
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a)

$$\frac{\partial^2 D_t(x,t)}{\partial x^2} = \frac{\rho}{G}\frac{\partial^2 D_t(x,t)}{\partial t^2} \qquad \left(v_t = \sqrt{\frac{G}{\rho}}\right)$$

$$\frac{\partial^2 D_l(x,t)}{\partial x^2} = \frac{\rho}{E}\frac{\partial^2 D_l(x,t)}{\partial t^2} \qquad \left(v_l = \sqrt{\frac{E}{\rho}}\right)$$

b) Enligt givna värden $v_t = 55{,}901\ldots$ m/s $\quad (v_l > v_t)$

$v_l = 136{,}936\ldots$ m/s

$\Delta t = 4$ s $\qquad \Delta x$?

$$\Delta t = t_t - t_l = \frac{\Delta x}{v_t} - \frac{\Delta x}{v_l} \Rightarrow$$

$$\Delta x = \frac{\Delta t}{\left(\frac{1}{v_t} - \frac{1}{v_l}\right)} = \frac{v_l \cdot v_t}{(v_l - v_t)} \cdot \Delta t \overset{\text{siffror}}{=} 0{,}377\ldots \text{ m}$$

Svar: $\Delta x = 0{,}4$ m

c) Cirkulär våg $\quad I = \frac{P_{av}}{s} \Rightarrow P_{av} = I \cdot s$

s ← vågfrontens omkrets

$$P_{av} = I_1 \cdot \cancel{2\pi} x_1 = I_2 \cdot \cancel{2\pi} x_2 \Rightarrow \frac{I_1}{I_2} = \frac{x_2}{x_1}$$

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2.

Intf term $2A_1A_2 \cdot \cos(\Delta\phi)$

Konstr. intf. då

$\Delta x_1 = 0{,}8$ m

$\Delta x_2 = 1{,}1$ m

$V = 340$ m/s

$$\Delta\phi = \frac{2\pi}{\lambda}\cdot\Delta x + \Delta\phi_0$$

a)
$$\left.\begin{aligned}\frac{2\pi}{\lambda}\cdot\Delta x_1 + \Delta\phi_0 &= 2\pi m \quad (m\in\mathbb{Z})\\ \frac{2\pi}{\lambda}\cdot\Delta x_2 + \Delta\phi_0 &= 2\pi(m+1)\end{aligned}\right\} \Rightarrow$$

$$\Rightarrow \frac{2\pi}{\lambda}(\Delta x_2 - \Delta x_1) = 2\pi \Rightarrow \lambda = \Delta x_2 - \Delta x_1 = \underline{\underline{0{,}3\text{ m}}}$$

$$f = \frac{V}{\lambda} = \frac{340}{0{,}3} = \underline{\underline{1133\text{ Hz}}}$$

b)
$$2\pi\frac{\Delta x_1}{(\Delta x_2 - \Delta x_1)} + \Delta\phi_0 = 2\pi m \Rightarrow \Delta\phi_0 = 2\pi\left(m - \frac{\Delta x_1}{(\Delta x_2-\Delta x_1)}\right)$$

$$\Delta\phi_0 = 2\pi\left(m - \frac{8}{3}\right)$$

$m=1$ ger $\Delta\phi_0 = -\frac{10}{3}\cdot\pi < -\pi$

$m=2$ ger $\Delta\phi_0 = -\frac{4}{3}\cdot\pi < -\pi$

$m=3$ ger $\underline{\underline{\Delta\phi_0 = \frac{2}{3}\cdot\pi}}$

$m=4$ ger $\Delta\phi_0 = \frac{8}{3}\pi > \pi$

Kdl av $\Delta\phi = \frac{2\pi}{0{,}3}\cdot\Delta x + \frac{2\pi}{3} = 2\pi\cdot m$ ger max vid 0,2 ; 0,5 ; 0,8 ; 1,1 m etc.

m=1, m=2, m=3, m=4

c) $\Delta\phi = \frac{2\pi}{0{,}3}\cdot\Delta x + \frac{2\pi}{3} = \underbrace{2\pi(m+\frac{1}{2})}_{\text{min}} \Rightarrow \Delta x = \frac{6m}{20} + \frac{1}{20}$

min då $0 < \Delta x < 1{,}1$ m?

$m=0$ ger $\Delta x = \frac{1}{20} = \underline{\underline{0{,}05\text{ m}}}$

$m=1$ ger $\Delta x = \frac{7}{20} = \underline{\underline{0{,}35\text{ m}}}$

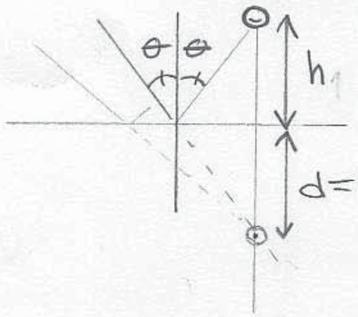
$m=2$ ger $\Delta x = \frac{13}{20} = \underline{\underline{0{,}65\text{ m}}}$

$m=3$ ger $\Delta x = \frac{19}{20} = \underline{\underline{0{,}95\text{ m}}}$

avstånd mellan
min 0,3 m
max 0,3 m
rimligt

min

a,

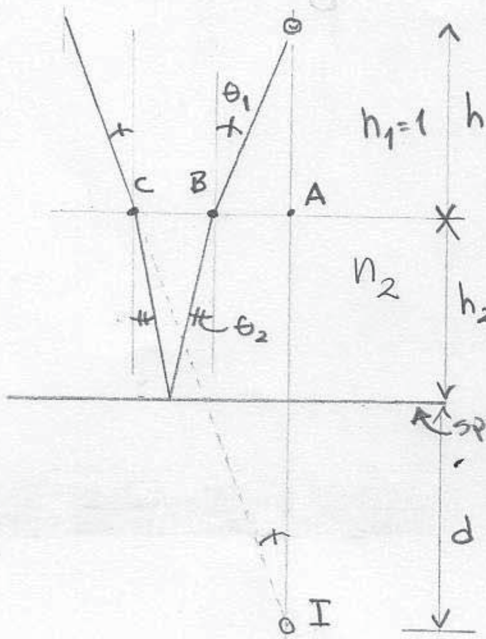


Svar: i) $d=h$

ii) Virtuell

↑
följer divergerande strålar bakåt.

b)



$n_1 \cdot \sin\theta_1 = n_2 \cdot \sin\theta_2 \Rightarrow \frac{\sin\theta_2}{\sin\theta_1} = \frac{n_1}{n_2}$

Sträcker i figuren

$|AB| = h_1 \cdot \tan\theta_1$

$|BC| = 2 \cdot h_2 \cdot \tan\theta_2$

$\underbrace{|AC|}_{|AB|+|BC|} = \underbrace{|AI|}_{h_2+d} \cdot \tan\theta_1$

$$\Rightarrow d = \frac{|AB|+|BC|}{\tan\theta_1} - h_2 =$$

$$= h_1 + 2h_2 \frac{\tan\theta_2}{\tan\theta_1} - h_2$$

Små vinklar: $\frac{\tan\theta_2}{\tan\theta_1} \approx \frac{\sin\theta_2}{\sin\theta_1} = \frac{1}{n_2}$ $\quad \therefore d \approx h_1 + h_2\left(\frac{2}{n_2} - 1\right)$

c) $d = 4,5 + 2,0\left(\frac{2}{1,333} - 1\right) = 5,50\ldots$ m

d) $\Delta d = d_v - d_{ny} = \cancel{h_1} + h_2\left(\underbrace{\frac{2}{n_v}}_{n_v = n_2} - \cancel{1}\right) - \cancel{h_1} - h_2\left(\frac{2}{n_{ny}} - \cancel{1}\right)$

$\Delta d = 33,6$ cm

$$\Rightarrow \Delta d = 2h_2\left(\frac{1}{n_v} - \frac{1}{n_{ny}}\right) \Rightarrow \frac{1}{n_{ny}} = \frac{1}{n_v} - \frac{\Delta d}{2h_2}$$

$\frac{1}{n_{ny}} = \frac{1}{1,333} - \frac{0,336}{2 \cdot 2,0}$ $\quad \therefore n_{ny} = 1,5010\ldots$

Kan vara bensen ($n = 1,5013$)

enl. PH T-4.3

(Alt. pyridin $n = 1,5094$)

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4.ab)

a, Antalet svängningar/sekund enligt figur
Mus: 29 Hz, Råtta 18 Hz, Marsvin 14 Hz, Huskatt 9 Hz
Pudel: 6 Hz, Labrador 5 Hz, Sumatratiger 4 Hz
Jättepanda 4 Hz, Brunbjörn 4 Hz

i, Huskatt (f) och Råtta ($2 \cdot f$)

ii, $f = 9\,Hz$

iii, $\omega = \sqrt{\frac{\kappa}{I}}$ (κ torsionskonst, I tröghetsmoment)

b, $T = 2\pi\sqrt{\frac{m'}{k'}}$ $\quad v_{max} = \omega \cdot A = \sqrt{\frac{k'}{m'}} \cdot A$

i, $T = 2\pi\sqrt{\frac{4m}{k}} = 2\pi\sqrt{\frac{m}{k}} \cdot 2$ $\quad v_{max} = \sqrt{\frac{k}{4m}} \cdot 10 = \sqrt{\frac{k}{m}} \cdot 5$

ii, $T = 2\pi\sqrt{\frac{4m}{k/2}} = \underline{\underline{2\pi\sqrt{\frac{m}{k}}\sqrt{8}}}$ $\quad v_{max} = \sqrt{\frac{k/2}{4m}} \cdot 20 = \sqrt{\frac{k}{m}} \cdot \frac{10}{\sqrt{2}}$

iii, $T = 2\pi\sqrt{\frac{2m}{k}} = 2\pi\sqrt{\frac{m}{k}}\sqrt{2}$ $\quad v_{max} = \sqrt{\frac{k}{2m}} \cdot 20 = \underline{\underline{\sqrt{\frac{k}{m}} \cdot \frac{20}{\sqrt{2}}}}$

iv, $T = 2\pi\sqrt{\frac{m}{k}}$ $\quad v_{max} = \sqrt{\frac{k}{m}} \cdot 5$

<u>Svar</u>: System ii, har längsta T
— " — iii, högsta v_{max}

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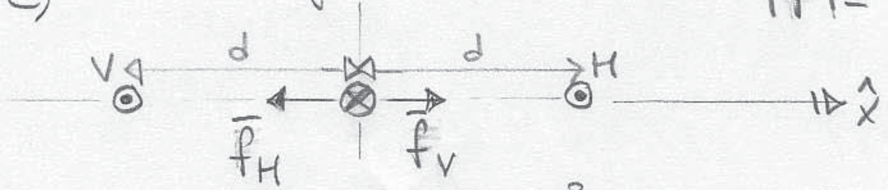
4.c)

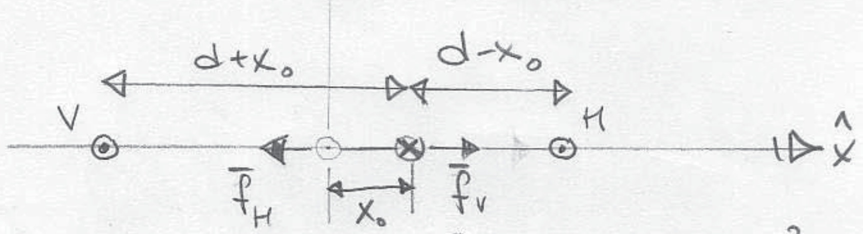
R.

c) Vid jämviktsläget: $|f| = \frac{\mu_0 I^2}{2\pi d}$ (N/m)

kraft per längdenhet



$$\bar{f} = \bar{f}_H + \bar{f}_V = \frac{\mu_0 I^2}{2\pi d}(-\hat{x}) + \frac{\mu_0 I^2}{2\pi d}(+\hat{x}) = \bar{0}$$



$$\bar{f} = \bar{f}_H + \bar{f}_V = \frac{\mu_0 I^2}{2\pi(d-x_0)}(-\hat{x}) + \frac{\mu_0 I^2}{2\pi(d+x_0)}(\hat{x}) = \frac{\mu_0 I^2}{2\pi}\left(\frac{1}{d+x_0} - \frac{1}{d-x_0}\right)\hat{x}$$

$$x_0 \ll d \quad \frac{1}{d+x_0} = d^{-1}\left(1+\frac{x_0}{d}\right)^{-1} = d^{-1}\left(1 - \frac{x_0}{d} + \ldots\right)$$

$$\frac{1}{d-x_0} = d^{-1}\left(1-\frac{x_0}{d}\right)^{-1} = d^{-1}\left(1+\frac{x_0}{d}+\ldots\right)$$

$$\frac{1}{d+x_0} - \frac{1}{d-x_0} \approx -2\cdot\frac{x_0}{d^2} \qquad \therefore \bar{f} \approx -\frac{\mu_0 I^2}{\pi d^2}\cdot x\,\hat{x}$$

per l.e.

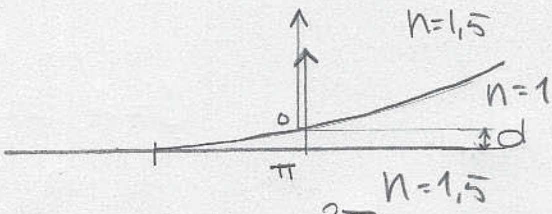
$$\left.\begin{array}{l} \bar{f}_{net} = \rho_l \cdot a_x \hat{x} \quad (\bar{F}_{net} = m\cdot\bar{a}) \\ \bar{f}_{net} = -\frac{\mu_0 I^2}{\pi d^2}\cdot x\,\hat{x} \end{array}\right\} \quad \underline{\underline{\ddot{x} + \frac{\mu_0 I^2}{\rho_l \pi d^2}\cdot x = 0}}$$

Harm. sv. rörelse

$$\omega = \sqrt{\frac{\mu_0 I^2}{\rho_l\cdot\pi d^2}} \qquad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\rho_l \pi d^2}{\mu_0 I^2}}$$

Svar: $\underline{\underline{T = 2\pi\sqrt{\frac{\rho_l\cdot\pi}{\mu_0}}\cdot\frac{d}{I}}}$

a)



$$\Delta\phi = k\cdot\Delta x + \Delta\phi_0 = \frac{2\pi}{\lambda}\cdot 2\cdot d + \pi$$

Konstr. int. då $\Delta\phi = 2\pi m$ $(m\in\mathbb{Z})$

$(m=1,2,3,\ldots)$

$$\Rightarrow \frac{2\cancel{\pi}}{\lambda}\cdot 2\cdot d + \cancel{\pi} = 2\cancel{\pi}\cdot m \Rightarrow 2d = \left(m-\tfrac{1}{2}\right)\lambda$$

alt. $2d = \left(m+\tfrac{1}{2}\right)\lambda$ $(m=0,1,2,3,\ldots)$

$$d = (2m+1)\frac{\lambda}{4} \quad (m=0,1,2,3,\ldots)$$

Enl. figur $R^2 = (R-d)^2 + r^2 \Rightarrow d = R - \sqrt{R^2-r^2}$

$$\Rightarrow \sqrt{R^2-r^2} = R-(2m+1)\cdot\frac{\lambda}{4} \Rightarrow \cancel{R^2}-r^2 = \cancel{R^2}-(2m+1)\frac{R\lambda}{2}+(2m+1)^2\frac{\lambda^2}{16}$$

$$\Rightarrow r = \sqrt{\frac{(2m+1)}{2}\cdot R\cdot\lambda - \frac{(2m+1)^2\lambda^2}{16}} \underset{(R\gg\lambda)}{\approx} \underline{\underline{\sqrt{\frac{(2m+1)}{2}\cdot R\cdot\lambda}}}$$

b) $m = \frac{r^2}{R\lambda} - \frac{1}{2} = \frac{(10\cdot10^{-3})^2}{5\cdot589\cdot10^{-9}} - \frac{1}{2} = 33{,}45\ldots \approx 33$

$m = 0,1,2,\ldots,33$ $\quad$ Svar: 34 st.

c) $k = \frac{2\pi}{\lambda_v} = \frac{2\pi}{\lambda}\cdot n_v$

$$m = \frac{r^2}{R\cdot\lambda}\cdot n_v - \frac{1}{2} = \frac{(10\cdot10^{-3})^2\cdot 1{,}333}{5\cdot589\cdot10^{-9}} - \frac{1}{2} = 44{,}763\ldots \approx 45$$

$m = 0,1,2,\ldots,45$ $\quad$ Svar: 46 st.

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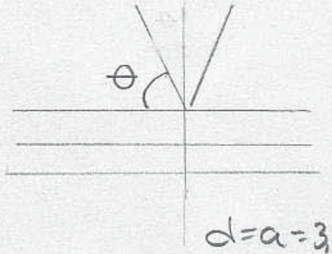
6.ade)

a, Bragg $2d \cdot \sin\theta = m\lambda$



$\sin\theta = \frac{m\lambda}{2d} \Rightarrow 2\theta = 2 \cdot \arcsin\left(\frac{m\lambda}{2 \cdot a}\right)$

$d = a = 3{,}36\,\text{Å}$

$m = 1$

$\lambda = 1{,}54\,\text{Å}$

$\therefore 2\theta = 26{,}496\ldots$ <u>Svar:</u> $2\theta = 26{,}5^\circ$

d,
i, $2\theta = 37{,}8^\circ$
ii, $2\theta = 61{,}6^\circ$
iii, $2\theta = 92{,}9^\circ$
iv, $2\theta = 111{,}4^\circ$

e, Ser ett mönster att $d = \frac{a}{\sqrt{b}}$
där b heltal relaterat till atomplanens "lutning"
jämfört med planen i uppg. a,
Om h anger antal steg i x-led till nästa atom i planet
och k — " — — " — i y-led — " — — " —
får vi $b = h^2 + k^2$ dvs. $d = \frac{a}{\sqrt{h^2+k^2}}$

<u>koll:</u> $a = \frac{a}{\sqrt{1^2+0^2}} = a$

i, $d = \frac{a}{\sqrt{1^2+1^2}} = \frac{a}{\sqrt{2}}$

ii, $d = \frac{a}{\sqrt{1^2+2^2}} = \frac{a}{\sqrt{5}}$

iii, $d = \frac{a}{\sqrt{1^2+3^2}} = \frac{a}{\sqrt{10}}$

iv, $d = \frac{a}{\sqrt{2^2+3^2}} = \frac{a}{\sqrt{13}}$

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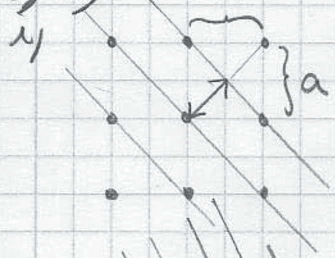
6.bc)

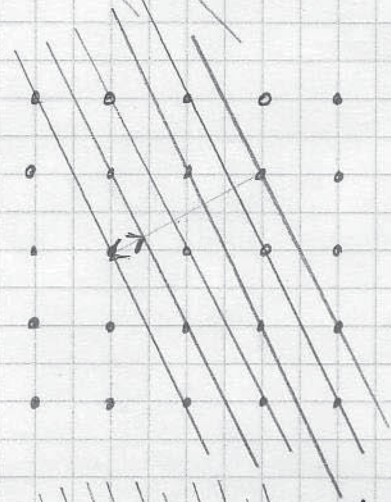
b) c)

i)



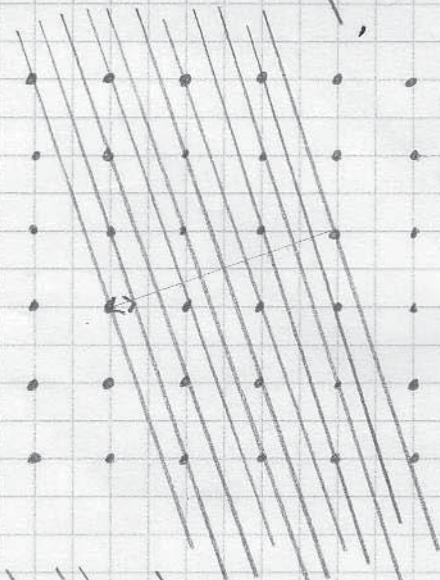
$$d = \frac{\sqrt{a^2 + a^2}}{2} = \frac{a}{\sqrt{2}} \approx 2{,}38\ \text{Å}$$

ii)



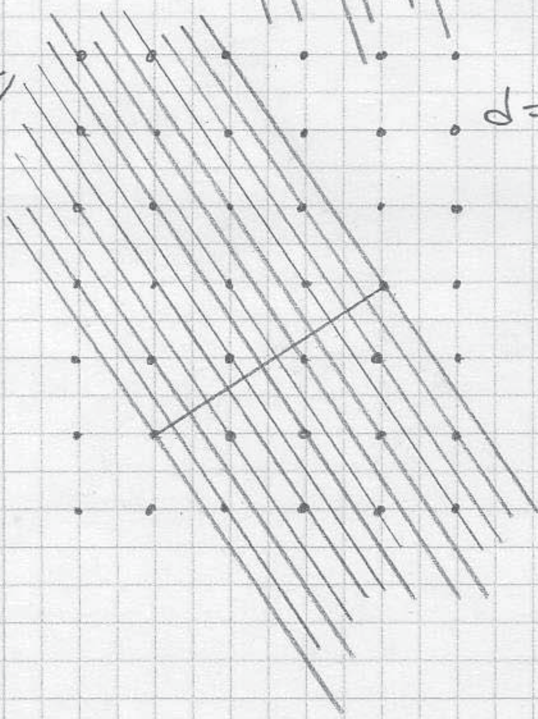
$$d = \frac{\sqrt{a^2 + 4a^2}}{5} = \frac{a}{\sqrt{5}} \approx 1{,}50\ \text{Å}$$

iii)



$$d = \frac{\sqrt{a^2 + 9a^2}}{10} = \frac{a}{\sqrt{10}} \approx 1{,}06\ \text{Å}$$

iv)



$$d = \frac{\sqrt{4a^2 + 9a^2}}{13} = \frac{a}{\sqrt{13}} \approx 0{,}93\ \text{Å}$$