1 Integral

Beräkna integralen $\int x \arcsin x \; dx$

$$\int x \arcsin x \; dx = \frac{x^2}{2} \arcsin x - \int \frac{x^2}{2} \frac{1}{\sqrt{1-x^2}} \; dx =$$

$$\left[x = \sin y, y = \arcsin x, \frac{dx}{dy} = \cos y, dx = \cos y \; dy \right]$$

$$\int \frac{\sin^2 y \cos y}{2\sqrt{1-\sin^2 y}} \; dy =$$

$$\frac{1}{2} \int \frac{\sin^2 y \cos y}{\sqrt{\cos^2 y}} \; dy$$

$$= \frac{1}{2} \int \sin^2 y \; dy = \frac{1}{2} \int \frac{1-\cos 2y}{2} \; dy$$

$$= \frac{1}{2} \int \frac{1}{2} \; dy - \frac{1}{2} \int \frac{\cos 2y}{2} \; dy =$$

$$\frac{1}{2}\frac{y}{2} - \frac{1}{2}\frac{\sin 2y}{4}$$

$$\frac{x^2}{2} \arcsin x - (\frac{y}{4} - \frac{\sin 2y}{8}) + C =$$

$$\left(not. \quad \sin 2t = 2 \sin t \cos t \right)$$

$$\frac{x^2}{2} \arcsin x - \frac{\arcsin x}{4} + \frac{2 \sin \arcsin x \cos \arcsin x}{8} + C =$$

$$= (\frac{x^2}{2} - \frac{1}{4}) \arcsin x + \frac{x}{4}\sqrt{1-x^2} + C \qquad (1)$$